DD MM AA

1

(2) Calcule el determinante de A si:

• i) A es matriz cuadrada involutiva?

$A = A^{-1}$

$\det(A) = \frac{1}{\det(A^{-1})} = \frac{1}{\det(A)} \Rightarrow \det^2(A) = 1$

$\det(A) = \sqrt{1} = \pm 1$

$\det(A) = \pm 1$

• ii) A es una matriz ortogonal

$A^{-1} = A^T$

$\det(A) = \frac{1}{\det(A^{-1})} = \frac{1}{\det(A)} = \frac{1}{\det(A)} \Rightarrow \det^2(A) = 1$

$\det(A) = \sqrt{1} = \pm 1$

$\det(A) = \pm 1$

• iii) A es una Matriz idempotente

$A^2 = A$

$\det(A^2) = \det(A) \cdot \det(A) = \det(A) \Rightarrow \det^2(A) = \det(A)$

$\det(A) = 1$ o $\det(A) = 0$

• iv) A es una Matriz antisimetrica

$A^T = -A$

$\det(A) = \det(-A^T) = (-1)^n \det(A^T)$

n es el numero de filas y columnas

Si n es impar entonse
$\det(A) = -\det(A)$ y entonces
$\det(A) = 0$

$$\det(A) = \begin{cases} 0 & \text{si } n \text{ es impar} \\ \det(A^T) & \text{si } n \text{ es par} \end{cases}$$

b) demuestre

i si A y B son matrices triangulares inferiores entonces el $\det(A \cdot B) = \det(A) \cdot \det(B)$

$$|A \cdot B| = \left| \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ 0 & b_{22} & \cdots & b_{2n} \\ 0 & 0 & \cdots & b_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{nn} \end{pmatrix} \right|$$

$$|A \cdot B| = \begin{vmatrix} a_{11}b_{11} & \sum_{k=1}^{2} a_{1k}b_{k2} & \cdots & \sum_{k=1}^{n} a_{1k}b_{kn} \\ 0 & a_{22}b_{22} & \cdots & \sum_{k=1}^{n} a_{2k}b_{kn} \\ 0 & 0 & \cdots & \sum_{k=1}^{n} a_{3k}b_{kn} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{nn}b_{nn} \end{vmatrix}$$

$$= a_{11}b_{11} \cdot a_{22}b_{22} \cdots a_{nn}b_{nn}$$

el determinante de A y de B es:

$$\det(A) = a_{11} \cdot a_{22} \cdots a_{nn}$$

$$\det(B) = b_{11} \cdot b_{22} \cdots a_{nn}$$

$$\det(A) \cdot \det(B) = a_{11} \cdot a_{22} \cdots a_{nn} \cdot b_{11} \cdot b_{22} \cdots a_{nn}$$

$$= \det(A \cdot B)$$

(c) en general no se cumple que $\det(A+B) = \det(A) + \det(B)$

Se demostrara esto con un ejemplo donde $\det(A+B) \neq \det(A) + \det(B)$

$$A = \begin{pmatrix} 4 & 1 \\ 2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} -3 & -1 \\ -2 & -1 \end{pmatrix} \qquad A+B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\det(A) = 6 \qquad \det(B) = 1 \qquad \det(A+B) = 1$$

$$\det(A) + \det(B) = 7 \neq \det(A+B)$$

A. lineal — Taller 2

1. (b) Demuestre

i. Sean A y B 2 matrices cuadradas de tamaño n:

$$|A| = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}a_{22}\cdots a_{nn}$$

$$|B| = \begin{vmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{vmatrix} = b_{11}b_{22}\cdots b_{nn}$$

$$|A||B| = a_{11}b_{11}a_{22}b_{22}\cdots a_{nn}b_{nn}$$

$$|AB| = \begin{vmatrix} a_{11}b_{11} & 0 & \cdots & 0 \\ a_{21}b_{11} + a_{22}b_{21} + \cdots + 0 & a_{22}b_{22} + \cdots + 0 & \ddots & 0 \\ a_{n1}b_{11} + a_{n2}b_{21} + \cdots + a_{nn}b_{n1} & a_{n2}b_{22} + \cdots + a_{nn}b_{n2} & \cdots & a_{nn}b_{nn} \end{vmatrix}$$

$$= a_{11}b_{11}a_{22}b_{22}\cdots a_{nn}b_{nn} \Rightarrow |AB| = |A||B| \quad //$$

ii. Sean A y B matrices cuadradas de orden n:

$$|A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = a_{11}C_{a11} + a_{12}C_{a12} + \cdots + a_{1n}C_{a1n}$$

$$|B| = \begin{vmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{vmatrix} = b_{11}C_{b11} + b_{12}C_{b12} + \cdots + b_{1n}C_{b1n}$$

$$|A+B| = \begin{vmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1}+b_{n1} & \cdots & a_{nn}+b_{nn} \end{vmatrix}$$

$$= (a_{11}+b_{11})C_{a+b11} + (a_{12}+b_{12})C_{a+b12} + \cdots + (a_{1n}+b_{1n})C_{a+b1n}$$

$|A|+|B| = (a_{11}(a_{11} + b_{11}(b_{11}) + (a_{12}(a_{12} + b_{12}(b_{12}) +$

$\ldots + (a_{1n}(a_{1n} + b_{1n}(b_{1n})$

Considerando lo anterior puede observarse que cuando los elementos de A tienden a ser positivos y también los de B puede que se observe lo siguiente:

$$|A+B| \neq |A| + |B|$$

por otro lado para elementos de A Y B específicos, puede que $|A+B| = |A| + |B|$. Lo que involucra que el cumplimiento o no de lo anterior, depende de los elementos de A y de B.

iii. Dado que A es triangular, se puede considerar así:

$$A = \begin{pmatrix} a_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

$$|A| = a_{11} \begin{vmatrix} a_{22} & \cdots & 0 \\ a_{21} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = a_{11} a_{22} \begin{vmatrix} a_{33} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

$$= a_{11} a_{22} a_{33} \begin{vmatrix} a_{44} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \cdots = a_{11} a_{22} \cdots a_{nn}$$

en vista de que $|A| = a_{11} a_{22} \cdots a_{nn}$ / basta con que un elemento de la diagonal principal sea igual a 0 para que $|A| = 0$ y por lo tanto $\det(A) \neq 0$ si $a_{11} a_{22} \cdots a_{nn} \neq 0$.

La propiedad anulativa de la multiplicación evidencia que si un elemento perteneciente a un producto de elementos es 0, el resultado de dicho producto también será 0.

Por la anterior propiedad si $a_{11} c_{11} a_{22} c_{22} \cdots a_{nn} c_{nn} = 0$
$\Rightarrow 0 = |A|$.

finalmente $|A| \neq 0 \Longleftrightarrow a_{11}, a_{22}, \cdots, a_{nn} \neq 0$ ||

iv. • $A = -A^t \Rightarrow A^t = -A$

• $|A^t| = |-A| = (-1)^n |A|$ dado que A es antisimétrica.

• Por propiedades de los determinantes:

$$|A| = |A^t| \Rightarrow |A^t| = |-A| = (-1)^n |A^t|$$

Como n es impar $(-1)^n = -1$, entonces:

$$|A| = -1|A|$$

$$|A| + |A| = -1|A| + |A|$$

$$2|A| = 0 \Rightarrow |A| = 0 \;//$$

$(a+b)(a^2+b^2) = (a^2+b^2)(a+b)$ //

V. $|A|$ $\xrightarrow{F_{n-1}-F_n,\ F_{n-2}-F_n,\ \cdots,\ F_1-F_n}$

$$\begin{vmatrix} 1 & 0 & 0 & \cdots & -1 \\ 0 & 1 & 0 & \cdots & -1 \\ 0 & 0 & 1 & \cdots & -1 \\ \vdots & \vdots & \vdots & & \vdots \\ X_1 & X_2 & X_3 & \cdots & 1+X_n \end{vmatrix} \xrightarrow{C_n + C_1 + C_2 + \cdots + C_{n-1}}$$

$$\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ X_1 & X_2 & X_3 & \cdots & 1+X_1+X_{n-1}+X_{n-2}+\cdots+X_2 \end{vmatrix}$$

En vista de que se trata de una matriz triangular inferior, el determinante estará dado por el producto de los elementos de la diagonal principal:

$$|A| = 1 + X_1 + X_2 + \cdots + X_n \;//$$

Vi. $|A| = 0 \longrightarrow$ dado que es singular

teniendo en cuenta la Expansión de Laplace del determinante se tiene:

$$A\,adj(A) = adj(A)A = \det(A)\,I$$

como $|A| = 0$ / $(A)Adj(A) = |A|I = [0]$ //

(c) $$|A| = \begin{vmatrix} \alpha & -3 \\ 4 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow \alpha(1-\alpha) + 4(3) = 0$$

$$-\alpha^2 + \alpha = -12$$

$$\alpha^2 - \alpha = 12$$

$$\alpha^2 - \alpha - 12 = 0$$

$$(\alpha + 3)(\alpha - 4) = 0 \Rightarrow \alpha_1 = -3,\ \alpha_2 = 4$$

(d)
$$|A| = \begin{vmatrix} -K & K-1 & K+1 \\ 1 & 2 & 3 \\ 2-K & K+3 & K+7 \end{vmatrix} = 0$$

$$|A| = -K\begin{vmatrix} 2 & 3 \\ K+3 & K+7 \end{vmatrix} - (K-1)\begin{vmatrix} 1 & 3 \\ 2-K & K+7 \end{vmatrix} + (K+1)\begin{vmatrix} 1 & 2 \\ 2-K & K+3 \end{vmatrix} = 0$$

$$|A| = -K(2K+14-3K-9) - (K-1)(K+7-6+3K) + (K+1)(K+3-4+2K) = 0$$

$$|A| = -K(-K+5) - (K-1)(4K+1) + (K+1)(3K-1) = 0$$

$$|A| = K^2 - 5K - 4K^2 - K + 4K + 1 + 3K^2 - K + 3K - 1 = 0$$

$|A| = 0 \Rightarrow$ No hay valor de K para el cual la matriz tenga inversa. Por lo tanto puede decirse que nunca tendrá inversa.

(e)
$$\begin{vmatrix} \cos\theta & \operatorname{Sen}\theta & 0 \\ -\operatorname{Sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta\begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} -$$

$$\operatorname{Sen}\theta\begin{vmatrix} -\operatorname{Sen}\theta & 0 \\ 0 & 1 \end{vmatrix} + 0 = \cos\theta(\cos\theta) - \operatorname{Sen}\theta(-\operatorname{Sen}\theta)$$

$= \cos^2\theta + \operatorname{Sen}^2\theta = 1 \Rightarrow |A| \neq 0 \Rightarrow$ tiene inversa

$$A^{-1} = \frac{1}{|A|}\operatorname{adj}(A)$$

$$\operatorname{Adj}(A) = C^t = \begin{pmatrix} \cos\theta & -\operatorname{Sen}\theta & 0 \\ \operatorname{Sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1}\begin{pmatrix} \cos\theta & -\operatorname{Sen}\theta & 0 \\ \operatorname{Sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5.

$$|A| = \begin{vmatrix} a+b & ab & 0 \\ 1 & a+b & ab \\ 0 & 1 & a+b \end{vmatrix} = (a+b)\begin{vmatrix} a+b & ab \\ 1 & a+b \end{vmatrix} -$$

$$ab\begin{vmatrix} 1 & ab \\ 0 & a+b \end{vmatrix} + 0 = (a+b)\left((a+b)^2 - ab\right) -$$

$$ab(a+b) = (a+b)^2 - ab(a+b) - ab(a+b)$$

$$= (a+b)^3 - 2ab(a+b)$$

$$= (a+b)\left((a+b)^2 - 2ab\right)$$

$$= (a+b)\left((a^2+2ab+b^2) - 2ab\right)$$

$$= (a+b)(a^2+b^2)$$

$$\Rightarrow (a+b)(a^2+b^2) = \frac{(a^4-b^4)}{(a-b)}$$

$$(a+b)(a^2+b^2) = \frac{(a^2+b^2)(a^2-b^2)}{(a-b)}$$

$$(a+b)(a^2+b^2) = \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)}$$

$$(a+b)(a^2+b^2) = (a^2+b^2)(a+b) \quad //$$

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2. Considere las siguientes matrices

$$A=\begin{pmatrix}-2 & 3 & 1\\ 4 & 6 & 5\\ 0 & 2 & 1\end{pmatrix} \quad |A| = -2\begin{vmatrix}-2 & 1\\ 4 & 5\end{vmatrix} + \begin{vmatrix}-2 & 3\\ 4 & 6\end{vmatrix} = -2(-10-4)+(12-12) = 4$$

$$B=\begin{pmatrix}4 & 1 & 1\\ 4 & 1 & 3\\ 4 & 2 & -1\end{pmatrix} \quad |B| = \begin{vmatrix}4 & 1 & 1\\ 4 & 1 & 3\\ 4 & 2 & -1\end{vmatrix} = \begin{vmatrix}4 & 1 & 1\\ 0 & 0 & 2\\ 4 & 2 & -1\end{vmatrix} = -2\begin{vmatrix}4 & 1\\ 4 & 2\end{vmatrix} = -2(8-4) = -8$$

$$C=\begin{pmatrix}-3 & 0 & 0 & 0\\ -4 & 7 & 0 & 1\\ 5 & 8 & -1 & 2\\ 2 & 3 & 0 & 6\end{pmatrix} \quad |C| = -3\begin{vmatrix}7 & 0 & 1\\ 8 & -1 & 2\\ 3 & 0 & 6\end{vmatrix} = -3[7(-6)+3] = 117$$

$$D=\begin{pmatrix}-2 & 0 & 0 & 7\\ 1 & 2 & -1 & 4\\ 3 & 0 & -1 & 5\\ 4 & 2 & 3 & 0\end{pmatrix} \quad |D| = -2\begin{vmatrix}2 & -1 & 4\\ 0 & -1 & 5\\ 2 & 3 & 0\end{vmatrix} - 7\begin{vmatrix}1 & 2 & -1\\ 3 & 0 & -1\\ 4 & 2 & 3\end{vmatrix} = -2\cdot[2\cdot(-5+4)-3(10)]$$

$$-7[(2)+2(9+4)-(0)] = 64+210 = 274$$

$$E=\begin{pmatrix}-5 & 6\\ 1 & 2\end{pmatrix} \quad |E| = -10-6 = -16$$

$$F=\begin{pmatrix}5 & -7\\ -6 & 18\end{pmatrix} \quad |F| = 90-42 = 48$$

- De ser posible calcular:

a) $\det(A^2 \cdot B) = \det^2(A)\cdot\det(B) = 4^2\cdot(-8) = -128$

b) $\det(A^{-1}BA) = \frac{1}{\det(A)}\cdot\det(B)\cdot\det(A) = \det(B) = -8$

c) $\det(C) = 117$

d) $\det(D) = 274$

e) $\det(\text{adj}(A))$

$$A^{-1} = \frac{\text{adj}^T(A)}{\det(A)} \Rightarrow \det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{\det^n(A)}\cdot\det(\text{adj}^T(A))$$

$$\frac{1}{\det(A)} = \frac{1}{\det^n(A)} \det(\text{adj}^T(A)) \Rightarrow \frac{\det^n(A)}{\det(A)} = \det(\text{adj}^T(A)) = \det(\text{adj}(A))$$

$$\det(\text{adj}(A)) = \det^{n-1}(A)$$

Respuesta

e) $\det(\text{adj}(A)) = \det^{3-1}(A) = \det^2(A) = 4^2 = 16$

f) $\det(\text{adj}(C)) = \det^{4-1}(C) = \det^3(C) = 117^3 = 1601613$

g) $\det(C^{-3}) = \det^3(C^{-1}) = \frac{1}{\det^3(C)} = \frac{1}{\det(\text{adj}(C))} = \frac{1}{1601613}$

h) $\det(E^2F^3) = \det^2(E) \cdot \det^3(F) = (-16)^2 \cdot (48)^3 = 7831552$

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3.

$$A = \begin{pmatrix} -2 & 3 & 1 \\ 4 & 6 & 5 \\ 0 & 2 & 1 \end{pmatrix} \qquad |A| = -2\cdot\begin{vmatrix} -2 & 1 \\ 4 & 5 \end{vmatrix} + \begin{vmatrix} -2 & 3 \\ 4 & 6 \end{vmatrix} = -2\cdot(-10-4) + (-12-12) = 28-24 = 4$$

$A_{11} = \begin{vmatrix} 6 & 5 \\ 2 & 1 \end{vmatrix} = 6-10 = -4$; $A_{12} = -\begin{vmatrix} 4 & 5 \\ 0 & 1 \end{vmatrix} = -4$

$A_{13} = \begin{vmatrix} 4 & 6 \\ 0 & 2 \end{vmatrix} = 8$; $A_{21} = -\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = -[3-2] = -1$

$A_{22} = \begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} = -2$; $A_{23} = -\begin{vmatrix} -2 & 3 \\ 0 & 2 \end{vmatrix} = -[-4] = 4$

$A_{31} = \begin{vmatrix} 3 & 1 \\ 6 & 5 \end{vmatrix} = 15-6 = 9$; $A_{32} = -\begin{vmatrix} -2 & 1 \\ 4 & 5 \end{vmatrix} = -[-10-4] = 14$

$A_{33} = \begin{vmatrix} -2 & 3 \\ 4 & 6 \end{vmatrix} = -12-12 = -24$

$$A^{-1} = \frac{1}{4}\begin{pmatrix} -4 & -1 & 9 \\ -4 & -2 & 14 \\ 8 & 4 & -24 \end{pmatrix} = \begin{pmatrix} -1 & -1/4 & 9/4 \\ -1 & -1/2 & 7/2 \\ 2 & 1 & -6 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 1 & 1 \\ 4 & 1 & 3 \\ 4 & 2 & -1 \end{pmatrix} \qquad |B| = \begin{vmatrix} 4 & 1 & 1 \\ 4 & 1 & 3 \\ 4 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 4 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & -2 \end{vmatrix} = -\begin{vmatrix} 4 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{vmatrix} = -(4\cdot 1\cdot 2) = -8$$

$B_{11} = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -1-6 = -7$; $B_{12} = -\begin{vmatrix} 4 & 3 \\ 4 & -1 \end{vmatrix} = -[-4-12] = 16$

$B_{13} = \begin{vmatrix} 4 & 1 \\ 4 & 2 \end{vmatrix} = 8-4 = 4$; $B_{21} = -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -[-1-2] = 3$

$B_{22} = \begin{vmatrix} 4 & 1 \\ 4 & -1 \end{vmatrix} = -4-4 = -8$; $B_{23} = -\begin{vmatrix} 4 & 1 \\ 4 & 2 \end{vmatrix} = -[8-4] = -4$

$B_{31} = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 3-1 = 2$; $B_{32} = -\begin{vmatrix} 4 & 1 \\ 4 & 3 \end{vmatrix} = -[12-4] = -8$

$B_{33} = \begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix} = 0$

$$B^{-1} = \frac{-1}{8}\cdot\begin{pmatrix} -7 & 3 & 2 \\ 16 & -8 & -8 \\ 4 & -4 & 0 \end{pmatrix} = \begin{pmatrix} 7/8 & -3/8 & -1/4 \\ -2 & 1 & 1 \\ -1/2 & 1/2 & 0 \end{pmatrix}$$

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$$C=\begin{pmatrix}-3&0&0&0\\-4&7&0&1\\5&8&-1&2\\2&3&0&6\end{pmatrix} \qquad |C|=\begin{vmatrix}-3&0&0&0\\-4&7&0&1\\5&8&-1&2\\2&3&0&6\end{vmatrix}=-3\begin{vmatrix}7&0&1\\8&-1&2\\3&0&6\end{vmatrix}=-3\cdot[7\cdot(-6)+1(3)]$$

$$=-3[-42+3]=117$$

$$C_{11}=\begin{vmatrix}7&0&1\\8&-1&2\\3&0&6\end{vmatrix}=7\begin{vmatrix}-1&2\\0&6\end{vmatrix}+\begin{vmatrix}8&-1\\3&0\end{vmatrix}=7(-6)+3=-39$$

$$C_{12}=-\begin{vmatrix}-4&0&1\\5&-1&2\\2&0&6\end{vmatrix}=-\left[-4\cdot\begin{vmatrix}-1&2\\0&6\end{vmatrix}+\begin{vmatrix}5&-1\\2&0\end{vmatrix}\right]=-(-4(-6)+2)=-26$$

$$C_{13}=\begin{vmatrix}-4&7&1\\5&8&2\\2&3&6\end{vmatrix}=(-4\cdot8\cdot6)+(7\cdot2\cdot2)+(1\cdot5\cdot3)-(2\cdot8\cdot1)-(3\cdot2\cdot(-4))$$

$$-(6\cdot5\cdot7)=-192+28+15-16+24-210$$

$$=-351$$

$$C_{14}=\begin{vmatrix}-4&7&0\\5&8&-1\\2&3&0\end{vmatrix}=-\left[-4\cdot\begin{vmatrix}8&-1\\3&0\end{vmatrix}-7\begin{vmatrix}5&-1\\2&0\end{vmatrix}\right]=-[-4\cdot3-7\cdot2]=26$$

$$C_{21}=\begin{vmatrix}0&0&0\\8&-1&2\\3&0&6\end{vmatrix}=0 \quad ; \quad C_{22}=\begin{vmatrix}-3&0&0\\5&-1&2\\2&0&6\end{vmatrix}=-3\begin{vmatrix}-1&2\\0&6\end{vmatrix}=-3\cdot(-6)=18$$

$$C_{23}=-\begin{vmatrix}-3&0&0\\5&8&2\\2&3&6\end{vmatrix}=-\left[-3\begin{vmatrix}8&2\\3&6\end{vmatrix}\right]=3\cdot[48-6]=126$$

$$C_{24}=\begin{vmatrix}-3&0&0\\5&8&-1\\2&3&0\end{vmatrix}=-3\cdot\begin{vmatrix}8&-1\\3&0\end{vmatrix}=-9 \quad ; \quad C_{31}=\begin{vmatrix}0&0&0\\7&0&1\\3&0&6\end{vmatrix}=0$$

$$C_{32}=-\begin{vmatrix}-3&0&0\\-4&0&1\\2&0&6\end{vmatrix}=0 \quad ; \quad C_{33}=\begin{vmatrix}-3&0&0\\-4&7&1\\2&3&6\end{vmatrix}=-3\begin{vmatrix}7&1\\3&6\end{vmatrix}=-3\cdot[42-3]=-117$$

$$C_{34}=\begin{vmatrix}-3&0&0\\-4&7&0\\2&3&0\end{vmatrix}=0 \quad ; \quad C_{41}=\begin{vmatrix}0&0&0\\7&0&1\\8&-1&2\end{vmatrix}=0$$

$$C_{42}=\begin{vmatrix}-3&0&0\\-4&0&1\\5&-1&2\end{vmatrix}=-3\cdot\begin{vmatrix}0&1\\-1&2\end{vmatrix}=-3\cdot1=-3$$

$$C_{43}=-\begin{vmatrix}-3&0&0\\-4&7&1\\5&8&2\end{vmatrix}=-\left[-3\cdot\begin{vmatrix}7&1\\8&2\end{vmatrix}\right]=3\cdot(14-8)=3\cdot6=18$$

$$C_{44} = \begin{vmatrix} -3 & 0 & 0 \\ -4 & 7 & 0 \\ 5 & 8 & -1 \end{vmatrix} = -3 \cdot \begin{vmatrix} 7 & 0 \\ 8 & -1 \end{vmatrix} = -3 \cdot (-7) = 21$$

$$C^{-1} = \frac{1}{117} \cdot \begin{pmatrix} -39 & 0 & 0 & 0 \\ -26 & 18 & 0 & -3 \\ -351 & 126 & -117 & 18 \\ 26 & -9 & 0 & 21 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{2}{9} & \frac{2}{13} & 0 & -\frac{1}{39} \\ -3 & \frac{14}{13} & -1 & \frac{2}{13} \\ \frac{2}{9} & -\frac{1}{13} & 0 & \frac{7}{39} \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{2}{9} & \frac{2}{13} & 0 & -\frac{1}{39} \\ -3 & \frac{14}{13} & -1 & \frac{2}{13} \\ \frac{2}{9} & -\frac{1}{13} & 0 & \frac{7}{39} \end{pmatrix}$$

$$D = \begin{pmatrix} -2 & 0 & 0 & 7 \\ 1 & 2 & -1 & 4 \\ 3 & 0 & -1 & 5 \\ 4 & 2 & 3 & 0 \end{pmatrix} \qquad \det(D) = 274$$

$$D_{1x1} = \begin{vmatrix} 2 & -1 & 4 \\ 0 & -1 & 5 \\ 2 & 3 & 0 \end{vmatrix} = 2 \begin{vmatrix} -1 & 5 \\ 3 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 4 \\ -1 & 5 \end{vmatrix} = 2 \cdot (-15) + 2(-5+4)$$
$$= -30 - 2 = -32$$

$$D_{12} = -\begin{vmatrix} 1 & -1 & 4 \\ 3 & -1 & 5 \\ 4 & 3 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 5 \\ 3 & 0 \end{vmatrix} + \begin{vmatrix} 3 & 5 \\ 4 & 0 \end{vmatrix} + 4 \begin{vmatrix} 3 & -1 \\ 4 & 3 \end{vmatrix} = -15 - 20 + 4 \cdot (9+4) = -17$$

$$D_{13} = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 0 & 5 \\ 4 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 5 \\ 2 & 0 \end{vmatrix} + 2 \begin{vmatrix} 3 & 5 \\ 4 & 0 \end{vmatrix} + 4 \begin{vmatrix} 3 & 0 \\ 4 & 2 \end{vmatrix} = -10 - 2(-20) + 4(6) = 54$$

$$D_{14} = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & -1 \\ 4 & 2 & 3 \end{vmatrix} = +3 \begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = +3(6+2) + (2-8) = 30$$

$$D_{21} = -\begin{vmatrix} 0 & 0 & 7 \\ 0 & -1 & 5 \\ 2 & 3 & 0 \end{vmatrix} = -\left[7 \begin{vmatrix} 0 & -1 \\ 2 & 3 \end{vmatrix} \right] = -7(2) = -14$$

$$D_{22} = \begin{vmatrix} -2 & 0 & 7 \\ 3 & -1 & 5 \\ 4 & 3 & 0 \end{vmatrix} = -2 \begin{vmatrix} -1 & 5 \\ 3 & 0 \end{vmatrix} + 7 \begin{vmatrix} 3 & -1 \\ 4 & 3 \end{vmatrix} = -2(-15) + 7(9+4) = 121$$

$$D_{23} = -\begin{vmatrix} -2 & 0 & 7 \\ 3 & 0 & 5 \\ 4 & 2 & 0 \end{vmatrix} = -\left[-2 \begin{vmatrix} 0 & 5 \\ 2 & 0 \end{vmatrix} + 7 \begin{vmatrix} 3 & 0 \\ 4 & 2 \end{vmatrix} \right] = -\left[-2(-10) + 7(6) \right] = -62$$

$$D_{24} = \begin{vmatrix} -2 & 0 & 0 \\ 3 & 0 & -1 \\ 4 & 2 & 3 \end{vmatrix} = -2 \begin{vmatrix} 0 & -1 \\ 2 & 3 \end{vmatrix} = -2(-2) = 4$$

$$D_{31} = \begin{vmatrix} 0 & 0 & 7 \\ 2 & -1 & 4 \\ 2 & 3 & 0 \end{vmatrix} = 7 \begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix} = 7(6+2) = 56$$

$$D_{32} = -\begin{vmatrix} -2 & 0 & 7 \\ 1 & -1 & 4 \\ 4 & 3 & 0 \end{vmatrix} = -\left[-2 \begin{vmatrix} -1 & 4 \\ 3 & 0 \end{vmatrix} + 7 \begin{vmatrix} 1 & -1 \\ 4 & 3 \end{vmatrix} \right] = -\left[-2(-12) + 7(3+4) \right] = -73$$

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$$D_{33} = \begin{vmatrix} -2 & 0 & 7 \\ 1 & 2 & 4 \\ 4 & 2 & 0 \end{vmatrix} = -2\begin{vmatrix} 2 & 4 \\ 2 & 0 \end{vmatrix} + 7\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = -2(-8) + 7(2-8) = -26$$

$$D_{34} = -\begin{vmatrix} -2 & 0 & 0 \\ 1 & 2 & -1 \\ 4 & 2 & 3 \end{vmatrix} = -\left[-2\begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix}\right] = 2\cdot(6+2) = 16$$

$$D_{41} = -\begin{vmatrix} 0 & 0 & 7 \\ 2 & -1 & 4 \\ 0 & -1 & 5 \end{vmatrix} = -\left[7\begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix}\right] = -7(-2) = 14$$

$$D_{42} = \begin{vmatrix} -2 & 0 & 7 \\ 1 & -1 & 4 \\ 3 & -1 & 5 \end{vmatrix} = -2\begin{vmatrix} -1 & 4 \\ -1 & 5 \end{vmatrix} + 7\begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix} = -2(-5+4) + 7(-1+3) = 16$$

$$D_{43} = -\begin{vmatrix} -2 & 0 & 7 \\ 1 & 2 & 4 \\ 3 & 0 & 5 \end{vmatrix} = -\left[-2\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} + 7\begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}\right] = -\left[-2(10) + 7(-6)\right] = 62$$

$$D_{44} = \begin{vmatrix} -2 & 0 & 0 \\ 1 & 2 & -1 \\ 3 & 0 & -1 \end{vmatrix} = -2\begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix} = 4$$

$$D^{-1} = \frac{1}{274}\cdot\begin{pmatrix} -32 & -14 & 56 & 14 \\ -17 & 121 & -73 & 16 \\ 54 & -62 & -26 & 62 \\ 30 & -4 & 16 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{16}{137} & -\frac{7}{137} & \frac{28}{137} & \frac{7}{137} \\ -\frac{17}{274} & \frac{121}{274} & -\frac{73}{274} & \frac{8}{137} \\ \frac{27}{137} & -\frac{31}{137} & -\frac{13}{137} & \frac{31}{137} \\ \frac{15}{137} & -\frac{2}{137} & \frac{8}{137} & \frac{2}{137} \end{pmatrix}$$

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$$E = \begin{pmatrix} -5 & 6 \\ 1 & 2 \end{pmatrix} \qquad \det(E) = -16$$

$$E^{-1} = -\frac{1}{16}\cdot\begin{pmatrix} 2 & -6 \\ -1 & -5 \end{pmatrix} = \begin{pmatrix} -\frac{1}{8} & \frac{3}{8} \\ \frac{1}{16} & \frac{5}{16} \end{pmatrix}$$

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$$F_{2\times 2} = i + (-2)^{i+j} = \begin{pmatrix} 5 & -7 \\ -6 & 18 \end{pmatrix} \qquad \det(F) = 48$$

$$F^{-1} = \frac{1}{48}\begin{pmatrix} 18 & 7 \\ 6 & 5 \end{pmatrix} = \begin{pmatrix} \frac{3}{8} & \frac{7}{48} \\ \frac{1}{8} & \frac{5}{48} \end{pmatrix}$$

4) Determine todos los valores de x para los cuales las siguientes matrices no singulares

a) $A = \begin{pmatrix} x & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$

$$|A| = \begin{vmatrix} x & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{vmatrix}$$

~~$|A| = 48 + (35x) + 24 -$~~

$|A| = 32x + 42 + 30 - (48 + 35x + 24)$

$|A| = 32x + 72 - (35x + 72)$

$|A| = 32x + 72 - 35x - 72$

$|A| = -3x \neq 0$

$|A| = x \neq 0$

$x \in \mathbb{R} - \{0\}$

b) $B = \begin{pmatrix} X & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & X \end{pmatrix}$

$$|B| = \begin{vmatrix} X & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & X \end{vmatrix}$$

$$\begin{matrix} X & 1 & 2 \\ 0 & 1 & 1 \end{matrix}$$

$$\begin{matrix} X & 1 & 2 \\ 0 & 1 & 1 \end{matrix}$$

$|B| = x^2 + 0 + 1 - (2 + 0 + 0)$

$|B| = x^2 - 2$

$|B| = x^2 - 2 \neq 0$

$x \neq \sqrt{2}$

$|B| = x^2 + 1 - 2$

$|B| = x^2 - 1$

$x^2 - 1 \neq 0$

$x \neq \sqrt{1}$

$x \neq 1, -1$

$x \in \mathbb{R} - x\{1, -1\}$

c) $C = \begin{pmatrix} x & 0 & 0 & 1 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 1 & 0 & 0 & x \end{pmatrix}$

$$|C| = \begin{vmatrix} x & 0 & 0 & 1 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 1 & 0 & 0 & x \end{vmatrix}$$

$|C| = x.x.x.x - x.x.0.0 - x.0.0.x + x.0.0.0$
$- 0.0.x.x + 0.0.0.x - 1.0.0.0 + 1.x.0.0$
$- 1.x.x.1 - 1.0.0.0$

$|c| = x^4 - x^2 \neq 0$ $\boxed{x^2 = u}$

$|c| = u^2 - u \neq 0$

$|c| = \begin{array}{l} x \neq 1 \\ x \neq -1 \\ x \neq 0 \end{array}$

$x \in \mathbb{R} - x\{-1, 0, 1\}$